On the motion of high energy wave packets and the transition radiation by "half-bare" electron

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Abstract

The problem of the motion of high-energy wave packets combined of free electromagnetic waves is considered. It is demonstrated that the transformation of such packets to the packet of spherically diverging waves happens on long distances along the packet's motion direction, that substantially exceed the radiated wavelength. The transition radiation by the "half-bare" ultrarelativistic electron is considered. It is demonstrated that the transition radiation by such an electron on the targets located inside and outside the coherence length of the radiation process would be substantially different.

Key words: equivalent photons method, wave packet, half-bare electron
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1. Introduction

Moving electron is the charge and the eigenfield (Coulomb field) moving together with it. Changing the electron's trajectory disturbs that field. The disturbance of the field could be treated as a packet of free plane electromagnetic waves. On large distances from the region where the acceleration had happened the packet transforms to the packet of diverging waves (the radiation field). For non-relativistic particles that happens on the distances of order of the length λ of radiated wave [1]. High energies make the stabilizing influence to wave packets that leads to a substantial increase of the length

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on which the packet's transformation takes place. This length could have macroscopic size, exceeding not only interatomic distances in matter, but also the size of the target and just the size of the experimental installation (detector). Hence it is important to know the behavior of such high-energy packets of electromagnetic waves in the region where that transformation happens. The present article is devoted to the examination of this problem.

Primarily, we consider the motion of Gaussian packet combined of plane waves with directions of the wave vector $\mathbf{k}$ close to each other. It is demonstrated that the shape of such a packet changes on the lengths that substantially exceed the wavelength $\lambda = 1/|\mathbf{k}|$ corresponding to the given absolute value of the wave vector $|\mathbf{k}|$.

Then we consider the motion of the wave packet that coincides in the time moment $t = 0$ with the eigenfield of the ultrarelativistic electron. It is demonstrated that the last packet also conserves its shape for a long time interval. Fourier component of this packet with the wavelength λ changes only on the distances z along the packet's direction of motion that exceed the length $2\gamma^2\lambda$, where γ is the electron's Lorentz factor. This length coincides with the coherence length of the radiation process of the relativistic electron $l = 2\gamma^2\lambda$ [2, 3].

The problem of special interest is the radiation under sharp (at the time moment $t = 0$) changing of the ultrarelativistic electron's velocity [3 - 5]. We demonstrate that the packets of electromagnetic waves arising in this case are close in their structure to the packets considered above. However, their manifestations in the direction of the initial and final motion of the electron are substantially different. Namely, on the distances $z < 2\gamma^2\lambda$, Fourier components with the wavelength λ of the packet moving along the direction of the initial electron motion will practically coincide with the Fourier components of the initial packet and, consequently, to the Fourier components of the Coulomb field of the electron moving in the initial direction without scattering. Oppositely, in the final electron's direction of motion, the field of the packet of free waves will screen the particle's eigenfield. The electron under such conditions was called in [4] as "half-bare particle", that is the particle whose specific Fourier components of the surrounding field are practically absent for a long time. We put attention to that the transition radiation by such particles and wave packets on the targets placed on the distances from the point of scattering larger and smaller than $2\gamma^2\lambda$ would be substantially different. The corresponding experiment would permit to observe direct manifestation of the "half-bare" electron and the process of

its dressing.

Let us note that for the charged particle the Gauss theorem is applicable, according to which the number of force lines of the electromagnetic field surrounding the electron does not change with time [1]. Under this the radiation process by electron can be presented as bending of these force lines [6 - 10]. Such a concept of radiation process relates to the complete electromagnetic field surrounding the electron. However, it does not contain such characteristics of the radiation process as coherence length and wave zone which are connected with determined Fourier components of this field. The term “half-bare electron” relates also to a determined Fourier component of the field surrounding the electron which is defined by the wavelength λ. So, the analysis of a space-time evolution of these Fourier components (wave packets) gives us a supplement for the picture of evolution of complete field surrounding the electron which is in accelerated motion.

We use the system of units in which the speed of light in vacuum is taken equal to the unit: $c = 1$.

2. Gaussian packet

The scalar potential of the packet of free electromagnetic waves could be expressed in the form of the following Fourier decomposition:

$$\varphi(\mathbf{r}, t) = \int \frac{d^3q}{(2\pi)^3} e^{i(\mathbf{qr}-qt)} C_q, \tag{1}$$

where C_q are the coefficients of the decomposition, $q = |\mathbf{q}|$. Consider at first the behavior of the packet combined at $t = 0$ of plane waves with the wave vectors $\mathbf{k}$ directed closely to some given direction (the z-axis). Supposing for simplicity that the distribution of the waves over directions of the vector $\mathbf{k}$ is Gaussian at $t = 0$, let us write the potential (1) in the form

$$\varphi_k(\mathbf{r}, 0) = \frac{1}{\pi\Delta^2} \int d^2\vartheta e^{-\vartheta^2/\Delta^2} e^{i\mathbf{kr}}, \tag{2}$$

where ϑ is the angle between $\mathbf{k}$ and the z-axis, and Δ^2 is the mean square value of the angle ϑ, $\Delta^2 \ll 1$. Coefficients $C_\mathbf{q}$ of a such packet have the form

$$C_\mathbf{q} = (2\pi)^3 \int \frac{d^2\vartheta}{\pi\Delta^2} e^{-\vartheta^2/\Delta^2} \delta(\mathbf{k} - \mathbf{q}), \tag{3}$$

where $\delta(\mathbf{k}-\mathbf{q})$ is the delta-function. In this case, according to (1),

$$\varphi_k(\mathbf{r},t) = \frac{1}{1+ikz\Delta^2/2}\exp\left\{ik(z-t) - \frac{(k\rho\Delta/2)^2}{1+i(kz\Delta^2/2)}\right\}, \tag{4}$$

where ρ is the transverse (in relation to the z-axis) component of $\mathbf{r}$.

Eq. (4) demonstrates that under $kz\Delta^2/2 \ll 1$

$$\varphi_k(\mathbf{r},t) \approx \exp\left\{ik(z-t) - (k\rho\Delta/2)^2\right\}, \tag{5}$$

and under the condition $kz\Delta^2/2 \gg 1$

$$\varphi_k(\mathbf{r},t) \approx -\frac{2i}{kz\Delta^2}\exp\left\{ik(z-t) + ik\frac{\rho^2}{2z} - \frac{\rho^2}{z^2\Delta^2}\right\}. \tag{6}$$

In the case $z \gg \rho$ the last formula could be written in the form of diverging wave:

$$\varphi_k(\mathbf{r},t) \approx -\frac{2i}{kr\Delta^2}\exp\left\{ik(r-t) - \frac{\rho^2}{z^2\Delta^2}\right\}, \tag{7}$$

where $r = \sqrt{\rho^2+z^2} \approx z + \rho^2/2z$.

So, on the distances z from the center of the packet that satisfy the condition

$$kz\Delta^2/2 \ll 1, \tag{8}$$

the shape of the packet (4) coincides with the packet's shape at $t=0$. Only on the distances z that satisfy the condition

$$kz\Delta^2/2 > 1, \tag{9}$$

the transformation of the packet of plane waves (4) into the packet of diverging spherical waves happens.

In the theory of radiation of electromagnetic waves, the spatial region where the field of moving charges acquires the form of spherically diverging waves, is called as *wave zone* (see, e.g. [1, 11]). Particularly, for non-relativistic charged particles the wave zone begins just on the distances from the radiating system that exceed the radiated wavelength (see [1]). Condition (9) demonstrates, however, that under $\Delta^2 \ll 1$ the formation of the wave zone takes place not on the distances $z > \lambda$, like in the problem of radiation of the non-relativistic particle, but on the distances

$$z > 2\lambda/\Delta^2, \tag{10}$$

which are much larger than the wavelengths $\lambda = 1/k$, of which the packet is composed (4). For small values of Δ^2 the length $z = 2\lambda/\Delta^2$ could reach macroscopic sizes.

3. Approximation of Coulomb field by the packet of plane waves

Such problem arises in the equivalent photons method (or the method of virtual quanta) when the Coulomb field of relativistic electron is replaced at some specific time moment ($t = 0$) by the packet of free electromagnetic waves. Indeed, the Fourier decomposition of the electron's Coulomb field could be written in the form

$$\varphi_c(\mathbf{r}, t) = \mathrm{Re} \int \frac{d^3k}{(2\pi)^3} e^{i(\mathbf{kr}-\mathbf{kv}t)} C_k^c, \tag{11}$$

where $\mathbf{v}$ is the electron's velocity directed along the z-axis, and

$$C_k^c = \frac{8\pi e \Theta(k_z)}{k_\perp^2 + k_z^2/\gamma^2}. \tag{12}$$

Here γ is the electron's Lorentz factor, k_z and $\mathbf{k}_\perp$ are the components of the vector $\mathbf{k}$, parallel and orthogonal to the z-axis, $\Theta(k_z)$ is the Heaviside's step function.

It is supposed in the equivalent photons method that at $t = 0$ the packet (1) composed of free electromagnetic waves coincides with the electron's Coulomb field moving with the velocity $\mathbf{v}$ [11 - 13]. That corresponds to Fourier decomposition (1) with the coefficients $C_q = C_k^c$.

For $\gamma \gg 1$ the main contribution to (1) would be made by the values $\mathbf{q} = \mathbf{k}$ which directions are close to the direction of the electron's velocity $\mathbf{v}$. Taking this into account, the packet (1) could be written in the form

$$\varphi(\mathbf{r}, t) = \mathrm{Re} \int_0^\infty dk \, \varphi_k(\mathbf{r}, t), \tag{13}$$

where

$$\varphi_k(\mathbf{r}, t) = \frac{2}{\pi} \exp\left[ik(z - t)\right] \int_0^\infty \frac{\vartheta d\vartheta}{\vartheta^2 + \gamma^{-2}} J_0(k\rho\vartheta) \, e^{-ikz\,\vartheta^2/2}. \tag{14}$$

Here ϑ is the angle between $\mathbf{k}$ and $\mathbf{v}$ ($\vartheta \ll 1$), and $J_0(x)$ is the Bessel function.

The function $\varphi_k(\mathbf{r}, t)$ has the same structure as the function (4) corresponding to Gaussian distribution of the vectors $\mathbf{k}$ over the angles ϑ. Namely, if $kz\vartheta^2/2 \ll 1$, the main contribution to the integral (14) is made by the values $\vartheta \sim \gamma^{-1}$ and

$$\varphi_k(\mathbf{r}, t) \approx \frac{2}{\pi} K_0(k\rho/\gamma) \, e^{ik(z-t)}, \tag{15}$$

where $K_0(x)$ is the modified Hankel function. In this case after integration over k in (13) we find that

$$\varphi(\mathbf{r},t) \approx \frac{e}{\sqrt{(z-t)^2+\rho^2/\gamma^2}}. \tag{16}$$

The main contribution to (13) is made by the values $k \sim \gamma/\rho$, hence Eq. (16) is valid in the range of coordinates ρ and z that satisfy the condition $z < \gamma\rho$. In this range of coordinates the packet under consideration moves with the velocity of light in the z-axis direction.

So, on the distances $z \lesssim 2\gamma^2\lambda$ the considered wave packet practically coincides with the initial one (at $t = 0$). Substantial transformation of the packet would happen only on the distances

$$z > 2\gamma^2\lambda. \tag{17}$$

In this case for the evaluation of the integral in (14) over ϑ one could apply the method of stationary phase. As a result of using of this method we find that

$$\varphi_k(\mathbf{r},t) = -\frac{2i}{\pi}\frac{1}{\vartheta_0^2+\gamma^{-2}}\frac{1}{kr}e^{ik(r-t)}, \tag{18}$$

where $r \approx z + \rho^2/2z$ and $\vartheta_0 = \rho/z$ is the point of stationary phase of the integral (14). We see that the components (18) of our packet have in the case under consideration the form of diverging spherical waves. Under this condition the angle ϑ_0 corresponds to the direction of radiation, and the function before the diverging wave describes the angular distribution of the radiation. So, the condition (17) draws out the wave zone in application to given problem.

The value $2\gamma^2\lambda$ presenting in the condition (17) is known in the theory of radiation by ultrarelativistic particles as the *formation length* or the *coherence length* [2, 3].

4. Transition radiation by a "half-bare electron"

High-energy packets of electromagnetic waves considered above manifest themselves in many problems connected with bremsstrahlung and diffraction radiation (see, e.g., [5, 14, 15]). Let us pay attention to some manifestations of such packets in the problem of transition radiation arising after sharp scattering of the high-energy electron on large angle.

The retarded solution for the potential of the electromagnetic field after the scattering of the electron at the time moment $t = 0$ on large angle could be expressed in the following form [3]:

$$\varphi(\mathbf{r}, t) = \Theta(r - t)\varphi_{\mathbf{v}}(\mathbf{r}, t) + \Theta(t - r)\varphi_{\mathbf{v}'}(\mathbf{r}, t), \tag{19}$$

where $\varphi_{\mathbf{v}}(\mathbf{r}, t)$ and $\varphi_{\mathbf{v}'}(\mathbf{r}, t)$ are potentials of the Coulomb field of the electrons moving all the time with the velocity $\mathbf{v}$ along the z-axis and with the velocity $\mathbf{v}'$ along the z'-axis, respectively. Eq. (19) demonstrates that after scattering of the electron at $t = 0$ its eigenfield strips out and after that transforms into the radiation field. In the direction of the final particle's motion the electron's eigenfield arises only in the region $r < t$ which is achieved by the signal about the scattering act at $t = 0$ (see Fig. 1, where the isolines of the scalar potential (19) are presented).

Consider the Fourier decomposition of (19):

$$\varphi(\mathbf{r}, t) = \frac{e}{2\pi^2} \operatorname{Re} \int \frac{d^3k}{k} e^{i\mathbf{kr}} \left\{ \frac{1}{k - \mathbf{kv}} e^{-ikt} \right.$$
$$\left. + \frac{1}{k - \mathbf{kv}'} \left[1 - e^{-i(k - \mathbf{kv}')t} \right] e^{-i\mathbf{kv}'t} \right\}. \tag{20}$$

The first term in this formula has the form of the packet of free waves moving along initial direction of the electron's velocity $\mathbf{v}$. This packet coincides with the electron's eigenfield at $t = 0$. According to (17), (18), the transformation of the Fourier components of this packet with the wavelength λ to the packet of diverging waves would happen on the distances $z > 2\gamma^2\lambda$. On smaller distances the packet of waves with the given value of $|\mathbf{k}|$ would be close to the initial one.

The length $l = 2\gamma^2\lambda$ on which the formation of the wave zone takes place could have macroscopic size. For example, for the electrons of energy 50 MeV in the range of wavelengths $\lambda \sim 10^{-1}$ cm this length is about 20 m (the measuring technique in such conditions is developed today — see, e.g. [15, 16]). So in the frames of that length one could arrange a thin target (see the target in Fig.1 which is arranged along the z-axis at $z < 2\gamma^2\lambda$) and examine the "transition radiation" of the considered packet (reflection of the waves, their passage through target etc.). The characteristics of such "transition radiation" practically would not differ from the characteristics of the transition radiation of the electron moving in the same direction (however, the electron in the packet under consideration is absent). But if the target

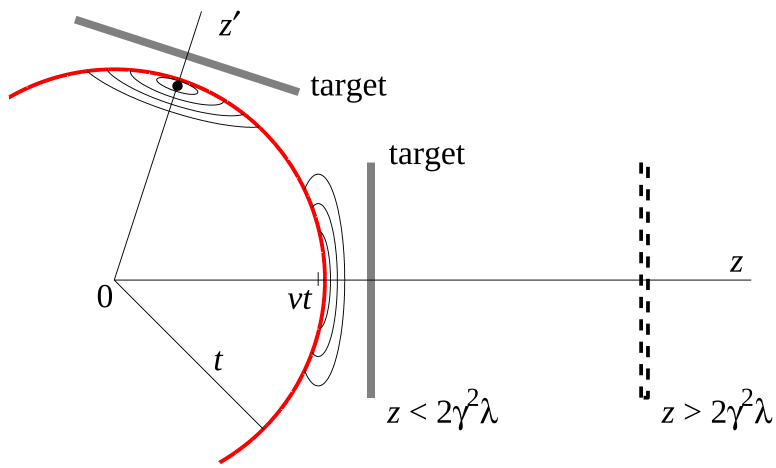

Figure 1: Equipotential surfaces of (19) and possible positions of targets for producing of the transition radiation.

would be located on the distance $z > 2\gamma^2\lambda$ (see dashed-line box in Fig.1), the features of the considered "transition radiation" would change due to the changing of the packet's shape (formation of the diverging waves).

The second term in (20) describes the field surrounding the electron after its scattering at $t = 0$, when its velocity became equal to $\mathbf{v}'$. This field consists of the electron's eigenfield moving with the velocity $\mathbf{v}'$ (the first term in square brackets in (20)) and the packet of free waves moving in the direction of $\mathbf{v}'$ coinciding at $t = 0$ with the opposite sign with Coulomb field of the electron (the second term in square brackets).

As it was demonstrated above, transformation of the packet of plane waves to the packet of diverging waves takes place on the distances $z' \sim 2\gamma^2\lambda$, where the axis z' is directed along $\mathbf{v}'$. During the time interval t over which the electron passes that distance, the substantial cancellation of the terms in the square brackets in (20) takes place. This mean that the electron stays on that distance in a "half-bare" state: the Fourier components with the wave vector $\mathbf{k}$ of its surrounding field would be suppressed comparing to the case $z' > 2\gamma^2\lambda$. Transition radiation of the electron with such field ("half-bare" electron) on the target located on the distance $z' < 2\gamma^2\lambda$ from the point of scattering (see the target on Fig.1 which is arranged along the z'-axis) would be suppressed in comparison to the case $z' > 2\gamma^2\lambda$.

The results obtained are correct for sharp scattering of an electron at a

large angle. Sharp scattering means that it takes place on the length which is much smaller than the coherent radiation length. At macroscopical values of the coherent length $l = 2\gamma^2\lambda$ it can occur not only under scattering of an electron by an atom but also under its scattering by a magnet. The only condition required for this is that the size of a scatterer was small as compared with the coherent radiation length.

Note that bremsstrahlung arising under collisions of the "half-bare" electron with the atoms of the medium located in the frames of the radiation formation length is suppressed comparing to the case when the collisions happen out of that length [5, 17, 18]. That lead, particularly, to such effects as Landau-Pomeranchuk-Migdal effect of suppression of the radiation by ultrarelativistic electrons in amorphous medium, the effect of suppression of the coherent bremsstrahlung in crystals and the effect of suppression of the radiation in thin layers of substance (see recent reviews and monographs [19 - 22] devoted to this topic, and the references therein). Experimental studies of these effects were carried out during last years and are made at present time on the accelerators of ultra high energies (see, e.g., [21 - 23]). Examination of the process of transition radiation by "half-bare" electron creates one more opportunity for study of manifestations of such an electron under its interaction with matter.

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References

[1] L.D. Landau, E.M. Lifshitz, The Classical Theory of Fields, Pergamon, Oxford, 1987.

[2] M.L. Ter-Mikaelyan, High-Energy Electromagnetic Processes in Condenced Media, Wiley, New York, 1972.

[3] A.I. Akhiezer, N.F. Shul'ga, high-energy Electrodynamics in Matter, Gordon and Breach Pub., Amsterdam, 1996.

[4] E.L. Feinberg, Sov. Phys. JETP 23 (1966) 132.

[5] N.F. Shul'ga, S.P. Fomin, Phys. Lett. A 114 (1986) 148.

[6] E.M. Purcell, Electricity and Magnetism, Berkeley Physics Course, v. 2. New York: McGraw-Hill, 1965.

[7] R.Y. Tsien, Am. J. Phys. 40 (1972) 46.

[8] H.C. Ohanian, Am. J. Phys. 48 (1980) 170.

[9] S.G. Arutunian, Sov. Phys. Usp. 150 (1986) 445.

[10] B.M. Bolotovskii, A.V. Serov Phys. Usp. 40 (1997) 1055.

[11] J.D. Jackson, Classical Electrodynamics, Wiley, New York, 1999.

[12] C. Weizsäcker, Z. Phys. 88 (1934) 612.

[13] E.J. Williams, Dan. Vid. Selsk. Mat. Phys. Medd. 13 (1935) 4.

[14] N.F. Shul'ga, S.N. Dobrovol'sky, JETP 90 (2000) 579; Nucl. Instrum. and Methods B 201 (2003) 123.

[15] G. Naumenko, X. Artru, A. Potylitsyn et al., `arXiv:0901.2630 [physics.acc-ph]`

[16] Y. Shibata, K. Ishi, T. Takahashi et al., Phys. Rev. E 49 (1994) 785.

[17] A.I. Akhiezer, N.F. Shul'ga, Sov. Phys. Usp. 30 (1987) 197.

[18] E.L. Feinberg, Sov. Phys. Usp. 22 (1979) 479.

[19] R. Blankenbecler, S. Drell, Phys. Rev. D 53 (1996) 6265.

[20] A.I. Akhiezer, N.F. Shul'ga, S.P. Fomin, Landau-Pomeranchuk-Migdal Effect, Cambridge Sci. Pub., 2005.

[21] S. Klein, Rev. of Mod. Phys. 71 (1999) 1501.

[22] U.I. Uggerhøj, Rev. of Mod. Phys. 77 (2005) 1131.

[23] H.D. Thomsen et al., Phys. Lett. B 672 (2009) 323.